Classifying the Heterogeneous Multi-Robot Online Search Problem into Quadratic Time Competitive Complexity Class

Shahar Sarid, Amir Shapiro, Elon Rimon, and Yael Edan

***Abstract*— We explore the problem where a group of robots with different velocities search for a target in an unbounded unknown environment. The target position is unknown, hence, an online search algorithm is developed. The *H-MRSTM* algorithm (*Heterogeneous Multi-Robot Search Time Multiplication*), launches a group of n robots from a common starting location to search for the target. The robots are assigned to search inside a series of concentric discs with increasing radii. Each robot is assigned to search inside a disc and when completing the search inside this disc without finding the target, the robot is assigned to search in the next unoccupied disc. We prove that every algorithm that solves this search problem must have at least a quadratic time competitive complexity and prove that the *H-MRSTM* algorithm's complexity is also quadratic. Hence, we obtain both an upper and lower bound on the time competitive complexity of the search problem. Consequently, *H-MRSTM* is proved to be optimal. Simulations in various environments show that the average case performance of *H-MRSTM* is superior to that of homogeneous multi-robot and single robot algorithms. In depth simulation analyses evaluated the effect of several other parameters such as the initial disc search time, the distribution of the velocities, the number of robots and the position of the target.**

I. Introduction

Finding a target whose position is unknown is an important problem in many applications (e.g., search and rescue, demining [1], planetary exploration missions [2] and surveillance [3]).

The basic motion planning [4] is what path a robot starting from a specific location should follow so that it will reach a known target point in a known environment. The solution is usually in the form of a path the robot should follow, avoiding obstacles in the environment. Mobile robot motion tasks which involve target finding include exploration, mapping, coverage, box pushing etc. A solution is called *off-line*, when the environment's geometry and the target position are known in advance, since all motions can be pre-computed prior to the actual execution of the mission. A solution is called *on-line* when no information about the environment is known in advance, as in a disaster area, or in dynamic environments such as offices and factories; the next step is computed while the robot moves and according to the information it receives from its sensors. In such cases, exploration tasks are useful to obtain information about the environment, and are usually accompanied by mapping. Such tasks incorporate world modeling requisites and highly regard communication capabilities. Moreover, the use of advanced sensors such as vision systems and laser scanners highly improves the performance of exploration algorithms, but may prove inefficient in worst case scenarios such as mazes, and congested environments, since in such cases these long range sensors become as effective as the touch sensors.

Exploration missions using short range sensors and target finding problems where the target position is unknown, and the robot can identify it only upon arrival, may be considered partially as area coverage problems. Area coverage missions are evident in real world problems such as vacuum cleaning, lawn mowing, painting, de-mining etc. The covering robot should pass a tool over all points of a defined area. A famous single robot Spanning Tree Covering algorithm (*STC*) [5] is a grid based coverage algorithm which constructs a spanning tree and deploys the robot to circumnavigate the tree thus covering the whole environment.

Many multi-robot grid-based coverage algorithms use or are compared with *STC* for performance measurement. Off-line algorithms such as [6], [7], cannot be used in on-line manner without major revisions.

Heterogeneous multi robot systems research is mainly focused on the cooperation of the different robots [8]. Coordination is composed of formation [9], task allocation [10], and integration [11]. Controlling swarms of mobile robots [12] is also considered to be in that category. Such problems focus on generality rather than specifically solving an exploration or a coverage problem. Other heterogeneous multi robot research direction include systems with human-robot interaction [13], such systems rely on human operator to partially or fully guide the robots during mission. Sensor networks are not originally related to robotics, however, sensor re-allocation [14], sensor repair by mobile robots [15] and mobile sensor agents [15], add the capability of movement to the sensors and therefore transform such problems to multi robot and sometimes to heterogeneous multi robot motion planning problems. In the aforementioned works, the sensors in the network are treated as multiple robots or as multiple targets. Sensors in such networks must have adequate ranges and specific positions in order to completely cover the area they are scattered in. They must form a network, and thus, must have communication capabilities.

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On this paper first the algorithm is developed and analyzed to behave optimally in worst case scenarios. hence, a computational competitive upper bound is yielded. A major part of the optimality proof is finding the lower bound of the problem. If the upper and lower bounds belong to the same functional class the problem can be classified into that time competitive complexity class. Then, simulation analysis tests the average case performance of the algorithm in various environments.

The structure and contributions of the paper are as follows. First, in the following section, the problem is defined and assumptions regarding it are presented. Next, the problem's lower bound is introduced. *H-MRSTM* algorithm is formally presented and its performance analyzed. Afterwards, the problem is classified into a time competitive complexity class, and *H-MRSTM* is proved optimal. In section VI the simulation analysis is presented and reinforces the analytical results. Finally, we conclude and discuss additional research directions and future work.

II. Problem's Definitions

The motion planning problem explored is defined as follows. A target must be found by a group of n robots in an unbounded, unknown two dimensional environment. The target position is unknown. The target can be identified using a specific target detector, mounted on each or on some of the robots. for example, a metal sensor for metal mines. The target is detected only when the robot is positioned directly above it. The size or diameter is D, a property important for the performance analysis. The robots are heterogeneous in their velocities, such that the slowest robot has a velocity v, and each robot j has a velocity $v_j = \beta_j \cdot v, \quad s.t. \quad \beta_j \geq 1$.

Definition 1 (Generalized Time Competitiveness [16]): An on-line algorithm solving a task P in time T is $f(t_{opt})$ *time competitive* when T is bounded from above by a scalable function $f(t_{opt})$ over all instances of P, and t_{opt} is the optimal off-line solution achieved while knowing all the information about the environment's geometry. In particular, $T \leq c_1 t_{opt} + c_0$ is the linear time competitiveness, while $T \leq c_2 t_{opt}^2 + c_1 t_{opt} + c_0$ is a quadratic time competitiveness, where the c_i's are positive constant coefficients that depend on the robot size D, the robots' velocity, the number of robots, and the geometry of the environment.

Note that the definition of $f(t_{opt})$ *time competitive* focuses on a particular algorithm solving the task P.

Further, the algorithm can be proved to be optimal if it can be shown that it performs in the bounds of the problem, if the problem can be classified into a competitive complexity class. Competitive complexity class is composed of two bounds, a universal lower bound which implies that no algorithm solving the problem can perform better than this bound, and an upper bound of an existing algorithm solving the aforementioned problem which is within the same performance function.

The following definition, which formalizes the last section, is based on [17],

Definition 2 (Time Competitive Complexity Class): A lower bound on the time competitiveness of a task P is a lower bound $T \geq g(t_{opt})$ over all on-line algorithms for P at worst case conditions, where T is the time it takes an algorithm to complete the task. If a time competitive upper bound $f(t_{opt})$ and a universal lower bound $g(t_{opt})$ for P are the same function up to constant coefficients, this function is the time competitive complexity class of P.

III. Competitive complexity lower bound

In this section a lower bound for the problem will be introduced. This lower bound proves that no algorithm can perform better than that bound. For this purpose a very "hard" environment, depicted in Fig. 1.(a) is used. The environment is circular and composed of radial corridors emanating from the start point S. Each part of the obstructed environment is further replaced with more corridors as seen in Fig. 1.(b). The width of the corridors is the same as the size of the robots, D. The target is placed in one of the corridors, in the farthest edge from S whose distance is marked R.

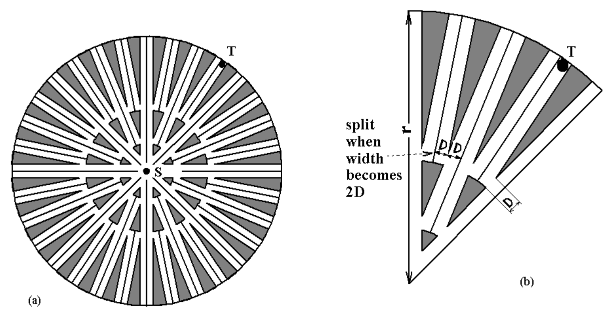

Fig. 1. [17] (a) The radial corridors environment. (b) Close up view of the environment.

Lemma 3.1 (Lower Bound): Any algorithm deploying n robots with different velocities, $v_j = \beta_j v, \ j = 1, \ldots, n$, $\beta_k \geq \beta_j \geq 1, \ \forall \ k > j$, and, $\beta_1 = 1$, in an unknown planar environment, recognizing the target only upon arrival, will find the target, within time T_{LB} which satisfies,

$$T_{LB} > \frac{2\pi v_n}{3Dn} t_{opt}^2$$

Proof: In order to find the target, every corridor must be inspected. Assuming the number of corridors is much greater than n, the number of robots, in worst case scenario, the robots will cover the whole free area of the environment at least once prior to reaching the target. This property is true for deterministic algorithms, since the target will be positioned in the last point the robots visited. If the algorithm is non-deterministic, at least one instance will produce this worst case behavior. By construction, the obstructed parts cover almost one third of the total environment area, and has a limit of one third as R goes to infinity. Hence, the free area equals $(2/3)\pi R^2$. Hence, the total path length equals $2\pi R^2/(3D)$. In the best scenario, no robot traverses the same path of any other robot, and the time it takes for all the robots to cover the whole environment satisfies $T_{LB} > \frac{2\pi R^2}{3Dnv_n}$,

where all the robots were considered to be fast, and therefore the strong inequality. The optimal off-line path length equals $l_{opt} = R + \epsilon'$, where the ϵ' is added due to the transitions between the corridors. Thus, the optimal solution satisfies $t_{opt} = (R + \epsilon')/(v_n)$. Hence, $T_{LB} > \frac{2\pi v_n}{3Dn} t_{opt}^2$ □

IV. H-MRSTM Algorithm

The *H-MRSTM*, a motion planning algorithm which uses multiple heterogeneous robots to search for a target whose position is unknown in an unknown environment is presented. The algorithm is based on a previous developed algorithm, *MRSAM* [18]. *H-MRSTM* is introduced and explained, along with a formal description of the algorithm.

In *H-MRSTM* the robots are heterogeneous in their velocities. *H-MRSTM* deploys each of its robots to search for the target in concentric discs with growing areas. Each robot searches for the target within a disc by covering it, the coverage algorithm should be efficient, e.g., *STC* algorithm [5]. After a robot finished searching for the target inside a disc, it moves to the next unoccupied search disc to search for the target in it if it did not find the target. The search is performed, again, by covering the reachable portions of the whole disc. Eventually, the search disc will contain a path to the target if it exists and it will be found. Though each consequent disc contains the previous one, the series of the discs' areas form a converging geometric series, thus yielding an upper bound on the path length and an optimal solution is obtained. The following conditions formalize the last idea.

Condition 1 (Search disc's area ratio): Each consequent search discs' area is greater than that of the previous disc.

During identical time periods, robots with different velocities will cover different areas, and the ratio of the covered areas will be identical to the velocities ratio. Consequently, a fast robot might finish covering the next search disc before the slow robot finished searching in the previous disc, thus, for *H-MRSTM*, condition 1 does not suffice, and the following condition complements it.

Condition 2 (Search time ratio): The time of search within each consequent search disc is greater than the time of search within the previous search disc.

H-MRSTM algorithm launches multiple robots from a common starting point *S* and assigns each robot j to a disc to search for the target *T* in it. All the discs are concentric and *S* is their center. n robots are deployed, and their velocities are $v_j = \beta_j v,\ j = 1, \ldots, n$, and $\beta_k \geq \beta_j \geq 1,\ \forall\ k > j$.

The first robot, R_1 is designated to search in the initial disc with search time T_0. Each of the following robots starts its search in a disc whose search time within is larger than the previous disc's search time by a factor of $\alpha_j > 1$. Each robot has its own multiplication factor $\alpha_j > 1$ according to its velocity, hence, the search times within the discs will be, $T_0, \alpha_2 T_0, \alpha_2\alpha_3 T_0, \alpha_2\alpha_3\alpha_4 T_0, ..., \prod_{i=2}^{n} \alpha_i T_0, \prod_{i=1}^{n} \alpha_i T_0,$ $\alpha_2 \prod_{i=1}^{n} \alpha_i T_0, \alpha_2\alpha_3 \prod_{i=1}^{n} \alpha_i T_0, ..., \prod_{i=2}^{n} \alpha_i \prod_{i=1}^{n} \alpha_i T_0, ...$

For example, in Fig. 2, *H-MRSTM* deploys a group of two robots to search for the target, robot 1 is initially assigned to search for the target inside a disc of search time T_0 and robot

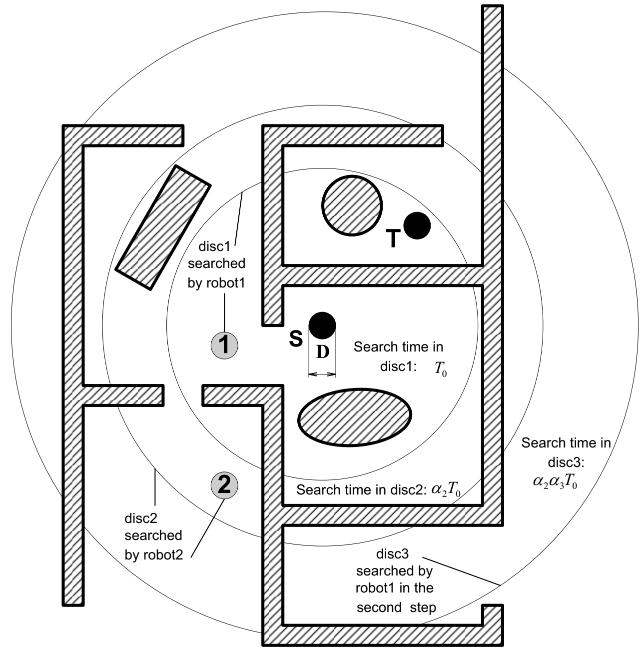

Fig. 2. A group of two robots launched by *H-MRSTM* searching the target.

R_2 is assigned to search inside a disc of search time $\alpha_2 T_0$. After robot 1 completes covering the entire portion of disc 1 which is accessible from *S* and fails to find the target, it starts searching for the target inside disc 3 of search time $\alpha_1\alpha_2 T_0$, in this case, robot R_1 will find the target while searching in disc 3, before or after robot R_2 completed searching in disc 2 and moved on to disc 4 of search time $\alpha_1\alpha_2^2 T_0$. Each robot searches for the target in the accessible portion of the disc allocated to it until the target is detected, or until the entire region accessible from *S* is explored without finding *T*. The search process in each disc is the same as in *MRSAM* [18]. Fig. 3 contains the pseudocode of *H-MRSTM* algorithm.

Implementation of the algorithm requires to address communication and on-board memory issues. At algorithm initialization, each robot must receive its own id number, and the following parameters, n, the total number of robots, and their velocities, β_j's. This way, each robot can calculate its own multiplication factor, α_j, and know in advance all the next discs in which it will be searching within. On termination, each robot must receive the stop signal. Thus, communication with and between the robots is not necessary during *H-MRSTM* execution. The memory requirements for each robot are as follows. Each robot must remember, its own id number, the number of robots, n, velocities of all the robots $(n-1)$, multiplication factors of all the robots $(n-1)$, and during execution, the current search disc number. Additional memory requirements depend on the coverage method within a search disc. For example, if *STC* [5] is being used, each search disc is decomposed into a grid of D-sized cells, and each robot builds a spanning tree on-line and circumnavigates it. If the number of unoccupied cells equals N, each robot must have a memory of size of $O(N)$ in order to execute *STC* [5]. Moreover, in worst case, N

Basic *H-MRSTM* Algorithm's Pseudocode
Sensors: A position sensor.
An obstacle detection sensor.
A target detection sensor.
Input: A start point *S* .
An initial search time T_0.
A group of n searching robots, with different velocities, $v_j = \beta_j v,\ i = 1, \ldots, n\ \ \beta_k \geq \beta_i \geq 1,\ \forall k > i$
Initialization:
For each robot $R_j,\ j = 1, \ldots, n$:
Set multiplication factor α_j.
Set initial search time $T_{1_{(R_j)}} = T_0 \quad , \ j = 1$
$T_{j_{(R_j)}} = \left(\prod_{i=2}^{j} \alpha_i\right) T_0,\ j \neq 1$
For each robot j,
Repeat:
Execute a *coverage tour* on the grid contained in the disc of search time T_j centered at $\mathcal{S}$. Scan each new free cell and its partially occupied neighbor cells for $\mathcal{T}$.
until one of the following occurs:
(1) The target is reached: STOP.
(2) If no new free cell is encountered during the current coverage tour:
STOP, the target is unreachable.
(3) Else, move to the next unoccupied disc k:
Set $T_{k_{(R_j)}} = (\prod_{i=1}^{n} \alpha_i)^{\left(\frac{k-j}{n}\right)} T_0 \quad , \ j = 1.$
Set $T_{k_{(R_j)}} = (\prod_{i=1}^{n} \alpha_i)^{\left(\frac{k-j}{n}\right)} \left(\prod_{i=2}^{j} \alpha_i\right) T_0,\ j \neq 1.$
End of Repeat loop

Fig. 3. *H-MRSTM* Pseudocode.

is quadratic in the number of cells composing the optimal off-line solution, l_{opt}.

V. Analytical performance analysis

In order to analyze *H-MRSTM*'s performance, several worst case scenario cases are inspected. Generally, the target is assumed to lie within a disc whose area is $A_{opt} = \pi l_{opt}^2$, where l_{opt} is the optimal off-line solution.

Proposition 5.1: If the target T is reachable, *H-MRSTM* finds the target using n robots and the travel time by robot j which found the target satisfies the quadratic inequality,

$$T_{R_j} < \frac{\pi \alpha_j \beta_n^2 v \left(\prod_{i=1}^{n} \alpha_i\right)}{\beta_{j-1} D \left(\prod_{i=1}^{n} \alpha_i - 1\right)} t_{opt}^2. \tag{1}$$

where D is the robot size, β_j is the ratio between the velocities of robot j and the slowest robot, α_j is the multiplication factor which is a function of n and of all the β's, and t_{opt} is the optimal off-line solution.

Proof: In the first case, R_n searched and totally covered a disc whose area is $A_{opt} - \epsilon$ and search time $T_{opt_n} - \epsilon$ and thus did not find the target. Consequently, R_1 is assigned afterwards to search for it in a disc whose search time is $\alpha_1 T_{opt_n}$(area is $\pi \alpha_1 l_{opt}^2$) and finds the target with time of

$$T_{i(R_1)} \leq \frac{\alpha_1 \pi (\beta_n v t_{opt})^2}{\beta_n v D} = \frac{\alpha_1 \pi \beta_n v}{D} t_{opt}^2, \tag{2}$$

assuming $T_{opt_n} = A_{opt}/(\beta_n v D)$, and $t_{opt} = l_{opt}/(\beta_n v)$.

The sum of the search times by R_1 is,

$$\begin{aligned} T_{R_1} \leq & T_1 + T_{1+n} + T_{1+2n} + \cdots + T_{i(R_1)} \\ = & T_0 + \left(\prod_{i=1}^{n} \alpha_i\right)^1 T_0 + \left(\prod_{i=1}^{n} \alpha_i\right)^2 T_0 + \cdots + \\ & + \left(\prod_{i=1}^{n} \alpha_i\right)^{\left(\frac{i-1}{n}\right)} T_0 \end{aligned} \tag{3}$$

The time series is a converging geometric series, which, its sum is,

$$T_{R_1} \leq \frac{\left(\prod_{i=1}^{n} \alpha_i\right)^{\left(\frac{i-1}{n}+1\right)} - 1}{\prod_{i=1}^{n} \alpha_i - 1} T_0 < \frac{\left(\prod_{i=1}^{n} \alpha_i\right)^{\left(\frac{i-1}{n}+1\right)}}{\prod_{i=1}^{n} \alpha_i - 1} T_0 \tag{4}$$

Comparing the two expressions for the time to cover the last disc, $T_{i(R_1)}$, (2) and (3), T_0 can be expressed in terms of i,

$$T_0 = \frac{\alpha_1 \pi \beta_n v}{D \left(\prod_{i=1}^{n} \alpha_i\right)^{\left(\frac{i-1}{n}\right)}} t_{opt}^2 \tag{5}$$

Substituting T_0 from (5) into (4) yields,

$$T_{R_1} < \frac{\left(\prod_{i=1}^{n} \alpha_i\right)^{\left(\frac{i-1}{n}+1\right)}}{\prod_{i=1}^{n} \alpha_i - 1} \frac{\alpha_1 \pi \beta_n v}{D \left(\prod_{i=1}^{n} \alpha_i\right)^{\left(\frac{i-1}{n}\right)}} t_{opt}^2.$$

Simplification yields,

$$T_{R_1} < \frac{\alpha_1 \pi \beta_n v \left(\prod_{i=1}^{n} \alpha_i\right)}{D \left(\prod_{i=1}^{n} \alpha_i - 1\right)} t_{opt}^2. \tag{6}$$

Repeating this process for cases where other robots reached the target leads to the generalization (1). □

Lemma 5.1: The time competitive complexity of *H-MRSTM* is minimal when for each of the n robots deployed, the multiplication factor equals,

$$\alpha_j = \frac{(n+1)^{1/n} \beta_{j-1}}{\prod_{i=1}^{n} \beta_j^{1/n}}, \beta_0 = \beta_n, \beta_1 = 1 \tag{7}$$

Proof: In order to find the optimal multiplication factors, $\alpha_i' s$, a new objection function which combines all the sums of times is formed,

$$\begin{aligned} T_{tot} &= T_{R_1} + T_{R_2} + \ldots + T_{R_n} \\ &= \frac{\pi \beta_n^2 v \left(\prod_{i=1}^{n} \alpha_i\right)}{D \left(\prod_{i=1}^{n} \alpha_i - 1\right)} \sum_{i=1}^{n} \frac{\alpha_i}{\beta_{i-1}} \end{aligned} \tag{8}$$

Differentiating T_{tot} (8) according to each of the $\alpha_i' s$, comparing each function to zero and finding the common roots yields, $\alpha_j = \frac{(n+1)^{1/n} \beta_{j-1}}{\prod_{i=1}^{n} \beta_i^{1/n}}, \beta_0 = \beta_n, \beta_1 = 1$ □

Theorem 1: The problem of on-line heterogeneous multi-robot navigation to an unknown target in an unknown environment belongs to the quadratic time competitive complexity class.

Proof: As defined in Definition 2, a time competitive complexity class is formed from a lower bound, and from an upper bound for the problem. In section III we found a lower bound for the research problem which is quadratic in the optimal off-line solution, t_{opt} (3.1). Then we presented *H-MRSTM* algorithm to that problem with an upper bound quadratic in t_{opt} (1). Thus, the research problem belongs to the quadratic time competitive complexity class. □

The following corollary asserts that if a path from the start S to the target T exist, *H-MRSTM* algorithm will find T.

Corollary 5.1: H-MRSTM is complete.

Proof: The first important property established in Proposition 5.1, is that if the target T is reachable, *H-MRSTM* will find it. The second property is that *H-MRSTM* will find the target in a finite and limited time and is deduced from the upper bound of the algorithm introduced in Proposition 5.1. The two properties implies the completeness of *H-MRSTM*. □

The following corollary asserts that *H-MRSTM* algorithm is optimal for the problem of finding a target with a group of heterogeneous robots.

Corollary 5.2: H-MRSTM is optimal.

Proof: In Theorem 1 we proved that the search problem belongs to the quadratic time competitive complexity class. *H-MRSTM* upper bound is time quadratic in the optimal off-line solution (1). Thus, *H-MRSTM* is optimal. □

VI. SIMULATION ANALYSIS

Simulations were conducted in a system designed especially for testing heterogeneous multi-robot algorithms. The effect of the following parameters on the algorithm's average case performance was tested: The number of robots, n, varies between 1 and 10 robots, where single robot simulations were conducted for comparison reasons; β_j indicate the velocities ratios among the robots within the group. β_j were calculated to meet the constraints derived from Condition 1 and Condition 2, $\alpha_j > 1, \beta_1 = 1, \beta_j > \beta_{j-1} \ \forall j > 1$, $\beta_{j-1} < \alpha_j \beta_j \ \forall j > 1, \ \beta_n < \alpha_1$; Three sets of β-s were tested for each run: 1) "beta-max": β_n with maximal value and the rest of the β-s close to 1, 2) "beta-linear": β-s with values distributed linearly from $\beta_1 = 1$ to the maximal possible β value, 3) "beta-homogeneous" : equal β-s; α_j, search time multiplication factors for robot j from a group of n robots are computed according to (7); Three different environments were evaluated: 1) free from obstacles, 2) "cave": composed of 7 rock like obstacles and is lightly congested (Fig. 4), 3) "vasche library floor1", is based on the 1st floor of the CSU Stanislaus library and is highly congested (Fig. 5). The two last data sets were obtained from the Robotics Data Set Repository (Radish) [19][1];

Ten different target positions were tested. Since the search path follows a spanning tree from one side, targets which are adjacent can be on two sides of the spanning tree and thus the path length to them can differ drastically. To overcome this problem, for each target position we further tested 8 adjacent target positions, surrounding the original position, such that it appears as a cluster of 9 target points. The ten target positions along with the optimal paths are shown in Fig. 5; The initial search radius varies between 3 and 40 with intervals of 4. A total of approximately 100,000 simulation runs were analysed (10 robots groups, 3 environments, 3 β-s sets, 13 initial radii, 10 target positions plus 8 surrounding targets positions).

The average time it took for a group of robots to reach a certain target is presented in Fig. 6. Beta 1 stands for homogeneous group of robots, all have β equals to 1. Beta L stand for linear distribution of β among the group. Beta M stands for one robot with maximal possible β and all the rest have β very close to 1. The graph depicted in Fig. 6 shows the average for all targets for all environments. Results indicate the following:

- Heterogeneous teams perform better than homogeneous teams and better than a single robot. Moreover, it seems

[1] Thanks to Richard Vaughan and to Ashley Tews for providing this data.

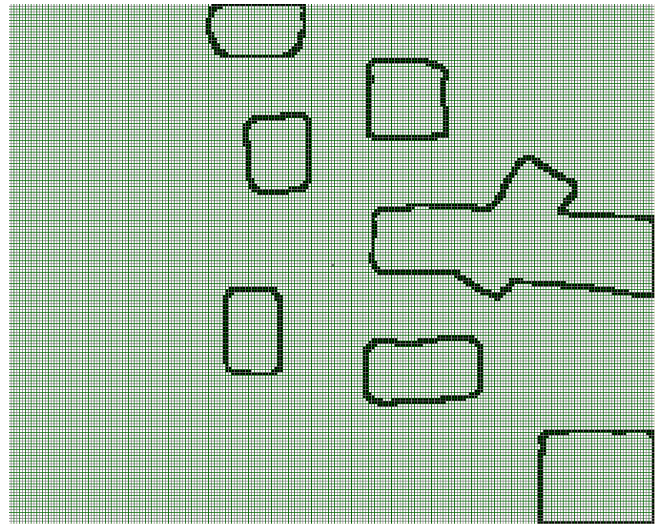

Fig. 4. *H-MRSTM* Cave environment.

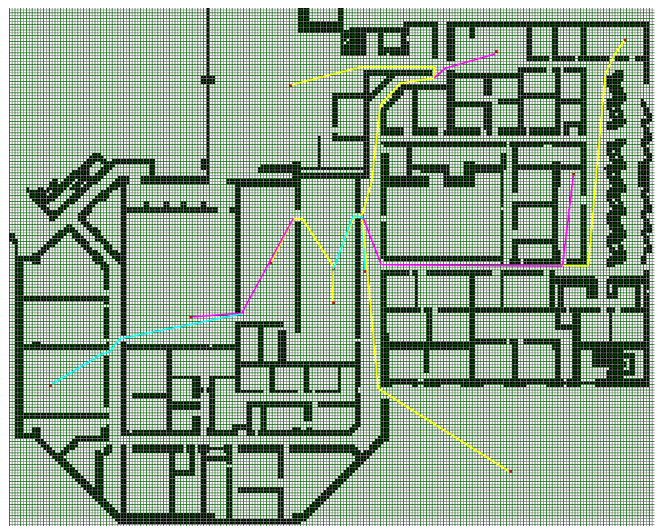

Fig. 5. 10 target positions in *H-MRSTM* "library" environment with optimal off-line paths marked.

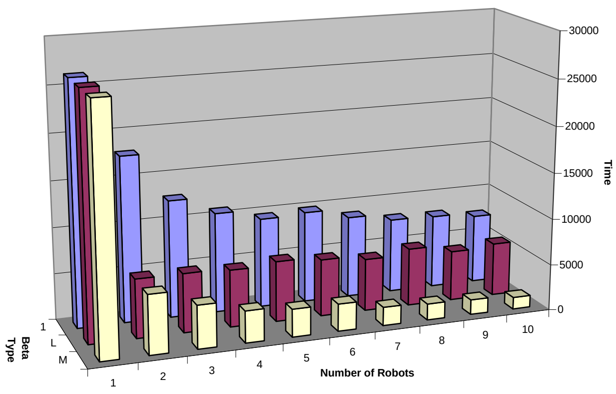

Fig. 6. Time average for all simulation runs. The units of the Time axis are D/v.

that ”beta-max” velocity distributed teams perform better than ”beta-linear” velocity distributed teams.

- As the number of robots grows, the time to find the target reduces.
- Reaching a target positioned in the same distance from the start, but in different environments, takes more time in less congested environments. For example, the average time to find the target in the ”cave” environment is greater than in the ”library” environment, since it has more free area, which, on average, the robots cover before finding the target.

VII. Conclusions and Future Work

The on-line motion planning problem of finding a target with unknown position by a group of heterogeneous mobile robots in an unknown planar environment was presented. A lower bound for the complexity of algorithms solving that problem was found to be quadratic in the optimal off-line solution. *H-MRSTM*, an algorithm solving that problem was introduced and its performance was analyzed and proved to be quadratic in the optimal off-line solution, too. The notions of *Time Competitiveness* and *Time Competitive Complexity Class* were formally defined. The search by a heterogeneous group of robots problem was classified into the quadratic time competitive complexity class. Consequently *H-MRSTM* was proved to be optimal. The superiority of *H-MRSTM* over homogeneous group of robots and over single robot was demonstrated in the simulation results.

Ongoing research aims to investigate the effect of uncertainty of the positioning system and of the sensors on the algorithm's performance by experimenting with a team of 2WD mobile robots, each equipped with an Arduino© microcomputer, 3 Daventech SRF05 ultra sonic rangefinders for obstacles detection, XBEE® wireless communication module to communicate with 12 Optitrack® IR cameras system to get position and orientation[2]. Performance analysis for heterogeneity of the covering tool (target detection sensor) size can be yielded from the relation between the covered area, A, the covering tool's size, D, and the equations of Section V. The following average case performance speedup is considered for further research. By keeping a common map between the robots, each robot can search only in the unvisited areas. This enhancement requires communication between the robots.

[2]Thanks to Lior Barshan, Hen Futerman, Lior Salem, and Guy Alon for their help in building the test bed.

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